Can Relic Superhorizon Inhomogeneities be Responsible for Large-Scale CMB Anomalies?

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We investigate the effects of the presence of relic classical superhorizon inhomogeneities during inflation. This superhorizon inhomogeneity appears as a gradient locally and picks out a preferred direction. Quantum fluctuations on this slightly inhomogeneous background are generally statistical anisotropic. We find a quadrupole modification to the ordinary isotropic spectrum. Moreover, this deviation from statistical isotropy is scale-dependent, with a $\sim -1/k^2$ factor. This result implies that the statistical anisotropy mainly appears on large scales, while the spectrum on small scales remains highly isotropic. Moreover, due to this $-1/k^2$ factor, the power on large scales is suppressed. Thus, our model can simultaneously explain the observed anisotropic alignments of the low-ℓ multipoles and their low power.

I. INTRODUCTION

The current observation data support the standard ΛCDM model [1]. Recently, however, there is an growing interest in analyzing possible large-scale anomalies of CMB, from both theoretical and observational sides [2],[3],[4],[6],[5],[7],[8],[11],[12][14].

It appears that the lowest CMB multipoles are anomalous in two seemingly distinct aspects. Firstly, the angular power C_ℓ at the lowest ℓ is abnormally suppressed [3]. Secondly, they have an improbable directionality revealed by the fact that for a certain preferred orientation, one m-mode absorbs most of the power, which may imply the presence of an "axis of evil" [4, 5].

The statistical properties of perturbations carry the same symmetries as the background on which they are generated. In the standard scenario, quantum fluctuations are assumed to be generated on a spatially homogeneous and isotropic background. Background spatial inhomogeneities are neglected in most of the analysis. Indeed, a long period of inflation pulls all non-smooth classical initial conditions out of the horizon. However, if inflation lasts the minimal number 60 e-folds or so, perturbations with comoving wave-numbers of cosmological interest cross the horizon just during the earliest several e-folds of inflation. Thus, the relic inhomogeneities may leave some marks in the primordial quantum fluctuations.

In this paper, we investigate the possible effects of a relic classical superhorizon inhomogeneity during inflation, especially its effects on the statistical properties of quantum fluctuations. In [6, 7], a single superhorizon perturbation mode were considered to explain the power asymmetry of CMB on large scales. While in this work, we consider relic superhorizon inhomogeneities as *background*. Perturbation theory in the presence of a spatially inhomogeneous inflaton background value has been investigated by several authors before [11],[12]. Quantum fluctuations on this slightly inhomogeneous background are generally statistical anisotropic.

The deviations from statistical isotropy can take on many forms, which may correspond to different physical origins. One simple form was presented in [4],$P(\boldsymbol{k}) = P(k)\left[1+g(k)(\hat{\boldsymbol{k}}\cdot\boldsymbol{n})^2\right]$. In this work, we find that the (leading-order) correction to the statistically isotropic power spectrum is of the ACW form [4], but with a k-dependent factor $g(k) \sim -1/k^2$. Thus, the power spectrum deviates from isotropy mainly on large scales (small k), while remains highly isotropic on small scales. Moreover, the spectrum itself is also suppressed on large scales. Thus, our model can simultaneously explain the observed anisotropic alignments of the low-ℓ multipoles and their low power.

II. MODEL AND BACKGROUND

In this work we investigate general single scalar field inflationary models, with an action of the form $S = \int d^4x\,\sqrt{-g}\,[R/2 + P(X,\phi)]$, where $X = -\frac{1}{2}(\partial\phi)^2$. Now we consider a slightly inhomogeneous inflaton background $\bar{\phi}(t,\boldsymbol{x})$. Firstly, we assume the deviation from homogeneity is very small, i.e., if $\bar{\phi}(t)$ is a spatial average of $\bar{\phi}(t,\boldsymbol{x})$ (e.g. in one Hubble volume), then we assume that

$$\left|\frac{\bar{\phi}(t,\boldsymbol{x})-\bar{\phi}(t)}{\bar{\phi}(t)}\right| \ll 1\,. \qquad (2.1)$$

Secondly, we assume the inhomogeneities are superhorizon. In other words, the typical comoving scale of these background inhomogeneities l is much larger than today's comoving Hubble scale, $l \gg (a_0H_0)^{-1}$. Actually, it has been known long before that inflation can occur in the presence of superhorizon initial inhomogeneities [9]. In other words, the inflaton field initially should be smooth up to physical scales larger than $\sim H^{-1}$. Thus, today's observational universe can indeed inflate from an initial small patch with classical inhomogeneities with typical comoving scale $l \gg (a_0H_0)^{-1}$.

Superhorizon inhomogeneities locally look like a gradient, $\bar{\phi}(t,\boldsymbol{x}) = \bar{\phi}(t,\boldsymbol{x}_0) + (\boldsymbol{x}-\boldsymbol{x}_0)\cdot\nabla\bar{\phi}(t,\boldsymbol{x}_0)+\cdots$. In this work, we neglect higher-order derivatives and treat $\nabla\bar{\phi}$ as approximately constant (while $\bar{\phi}$ itself is indeed slow-rolling, for instance).

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III. LINEAR PERTURBATIONS

We split $\phi(t,\boldsymbol{x})$ into background and fluctuation configurations,

$$\phi(t,\boldsymbol{x}) = \bar{\phi}(t,\boldsymbol{x}) + \delta\phi(t,\boldsymbol{x}) \,. \tag{3.1}$$

In the following we denote $\partial_i\bar{\phi}(t,\boldsymbol{x}) = A_i$, which we have assumed to be constant.

As a first-step investigation, we neglect the metric perturbations. Thus the calculation is straight forward since all we have to do is to expand the action for the scalar field directly. According to (3.1), simple Taylor expansion of $P(X,\phi)$ around the background $\bar{\phi}$ up to second order of $\delta\phi$ gives $-\frac{1}{2}\Sigma^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi - P_{,X\phi}\partial^\mu\bar{\phi}\partial_\mu\delta\phi\,\delta\phi + \frac{1}{2}P_{,\phi\phi}\delta\phi^2$, where we have defined

$$\Sigma^{\mu\nu} \equiv P_{,X}g^{\mu\nu} - P_{,XX}\partial^\mu\bar{\phi}\partial^\nu\bar{\phi} \,. \tag{3.2}$$

Note that due to the non-vanishing background gradient $\partial_i\bar{\phi}$, Σ^{ij} is not proportional δ_{ij} any more. The presence of the additional term which is proportional $\partial_i\bar{\phi}\partial_j\bar{\phi}$ in Σ^{ij} breaks spatial rotational invariance and will be responsible for the generation of statistical anisotropies of the scalar perturbations $\delta\phi$.

After introducing a new variable $u(\eta,\boldsymbol{x}) = a\sqrt{-\Sigma^{00}}\delta\phi$, the second-order action for the scalar field perturbations can be written as

$$S = \int d\eta \frac{d^3k}{(2\pi)^3}\frac{1}{2}\left[u'_{-\boldsymbol{k}}u'_{\boldsymbol{k}} + \frac{2i\sqrt{\epsilon}\gamma}{aH}(\boldsymbol{A}\cdot\boldsymbol{k})u'_{-\boldsymbol{k}}u_{\boldsymbol{k}} - \left(c^2k^2 - \frac{a''}{a} + \mathcal{M} + \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{a^2H^2}\right)u_{-\boldsymbol{k}}u_{\boldsymbol{k}}\right] , \tag{3.3}$$

where η is comoving time, and a prime represents $\partial/\partial\eta$, $\mathcal{H} \equiv a'/a$, and $\mathcal{M}$ is an effective mass term, ϵ is defined as $\epsilon = \dot{\phi}^2/H^2$. Here we define two dimensionless parameters

$$c^2 = \frac{P_{,X}}{-\Sigma^{00}} \,, \qquad \gamma = \frac{H^2 P_{,XX}}{\Sigma^{00}} \,. \tag{3.4}$$

In this paper, we assume $\gamma > 0$. Note that the canonical case corresponds to $c^2 = 1$ and $\gamma = 0$. In deriving (3.3), we use the approximation that $\Sigma^{\mu\nu}$ is a function of only η, that is, we neglect the spatial dependence of $\Sigma^{\mu\nu}$.

A. Equation of Motion for the Perturbations and Solutions

The classical equation of motion for the mode function according to the second-order action (3.3) is

$$u''_{\boldsymbol{k}} + \frac{2i\sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{aH}u'_{\boldsymbol{k}} + \left(c^2k^2 - \frac{a''}{a} + \mathcal{M}a^2 + \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{a^2H^2}\right)u_{\boldsymbol{k}} = 0 \,. \tag{3.5}$$

As it can be seen, an anisotropic dispersion relation arises, which will be responsible for the generation of a statistically anisotropic power spectrum.

Now we take the scale factor as $a(\eta) = -1/H\eta$. Remarkably, in this simplest case, equation (3.5) has an analytic solution,

$$u_{\boldsymbol{k}}(\eta) = \frac{\Gamma(\alpha-\nu)}{2^{\nu+1}\sqrt{\pi}}e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}e^{\frac{i}{2}\lambda\eta^2}\sqrt{-\eta}(-ck\eta)^\nu U(\alpha,\nu+1,z) \,, \tag{3.6}$$

in which $U(\alpha,\nu+1,z)$ is the confluent hypergeometric function, with

$$\begin{aligned}
\nu &= \sqrt{\frac{9}{4} - \frac{\mathcal{M}}{H^2}} \,, \\
\alpha &= \frac{1}{2}(\nu+1) - \frac{ic^2k^2 - \sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{4|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}} \,, \\
z &= -i|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}\eta^2 \,, \\
\lambda &= |\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2} + \sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k}) \,.
\end{aligned} \tag{3.7}$$

Due to the presence of the factor $\boldsymbol{A}\cdot\boldsymbol{k}$, the modes generally depend on $\boldsymbol{k}$ rather than $k = |\boldsymbol{k}|$. Here the coefficient in (3.6) is chosen in order to satisfy the Wronskian normalization condition $u'_{\boldsymbol{k}}(\eta)u^*_{\boldsymbol{k}}(\eta) - u'^*_{\boldsymbol{k}}(\eta)u_{\boldsymbol{k}}(\eta) = i$. In the limit $A_i \to 0$, it can be verified that (3.6) reduces to the well-known functional form $u_{\boldsymbol{k}}(\eta) \xrightarrow{A_i\to 0} \frac{\sqrt{\pi}}{2}e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}\sqrt{-\eta}H^{(1)}_\nu(-ck\eta)$, which describes nothing but the normalized mode function of a massive scalar field in pure de Sitter spacetime. Thus our mode solution (3.6) generalizes the standard homogeneous and isotropic background to the case of the presence of superhorizon background inhomogeneities.

B. Anisotropic Power Spectrum

When all modes of cosmological interest exit the Hubble scale, that is, in $\eta \to 0$ limit, since $U(\alpha,\beta,z) \to \frac{\Gamma(\beta-1)}{\Gamma(\alpha)}z^{1-\beta}$ as $z \to 0$ (when $\beta > 2$), we get

$$u_{\boldsymbol{k}}(\eta) = \mathcal{A}(\boldsymbol{k})\, e^{\frac{i\pi}{2}(\nu-\frac{1}{2})}2^{\nu-\frac{3}{2}}\frac{\Gamma(\nu)}{\Gamma(3/2)}\frac{1}{\sqrt{2ck}}\,(-ck\eta)^{\frac{1}{2}-\nu} \,, \tag{3.8}$$

where we have defined an anisotropic deformation factor,

$$\mathcal{A}(\boldsymbol{k}) \equiv \frac{\Gamma(\alpha-\nu)}{\Gamma(\alpha)}\left(\frac{4i|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}}{c^2k^2}\right)^{-\nu} \,, \tag{3.9}$$

which is responsible for the anisotropic deformation of the power spectrum on large scales. Here α,ν are given in (3.7). In the standard senarios, $\mathcal{A}(\boldsymbol{k})$ is just $\mathcal{A}(\boldsymbol{k}) = 1$. In our case, $\mathcal{A}$ depends on $\boldsymbol{k}$, not only on its amplitude k, but also on it direction, more precisely, on $\hat{\boldsymbol{A}}\cdot\hat{\boldsymbol{k}}$ (see FIG. 1).

We would like to investigate the leading order effect of $A_i = \partial_i\bar{\phi}$. Takeing the limit $A_i \to 0$ in (3.9) and using Stirling's formula, we get

$$\mathcal{A}(\boldsymbol{k}) = 1 - i\frac{\nu\sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{c^2k^2} - \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{c^4k^4}\left[\frac{2}{3}\nu(\nu^2-1) + \frac{\nu(\nu+1)(4\nu-1)}{6}\epsilon\gamma\right] . \tag{3.10}$$

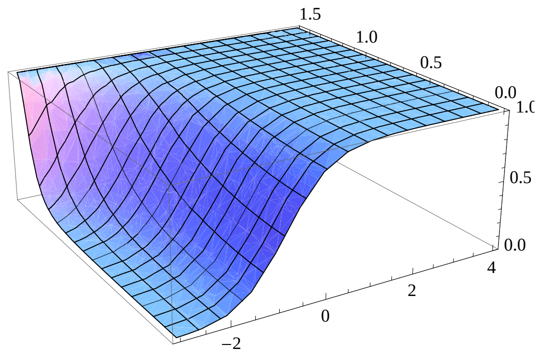

FIG. 1: Deformation factor $|\mathcal{A}(\boldsymbol{k})|^2$ as function of $k = |\boldsymbol{k}|$ and θ, where $\cos\theta \equiv \hat{\boldsymbol{A}} \cdot \hat{\boldsymbol{k}}$. Parameters are chosen as $c = 2$, $A = \gamma = 1$, $\epsilon = 0.01$, $\nu = 1.52$. We plot $\ln k$ from -3 to 4, and θ from 0 to $\pi/2$.

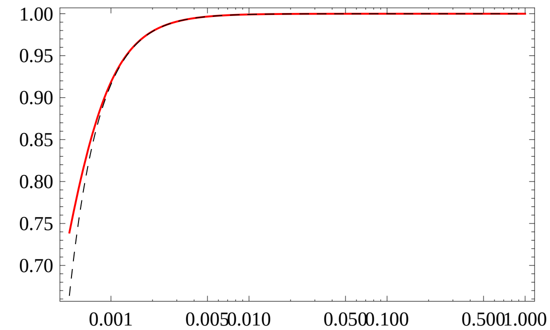

FIG. 2: Leading-order quadrupole terms of $|\mathcal{A}(\boldsymbol{k})|$ (for a given direction of $\hat{\boldsymbol{k}}$). The black dashed curve is the exact result, the red curve corresponds to the quadrupole term given in (3.10). Here we have chosen that $\boldsymbol{k}$ is parrallel to $\boldsymbol{A}$ without loss of generality. Parameters are chosen as $A = 10^{-3}$, $c = 2, n_s = 0.96$, $\epsilon = 0.01$. We plot k from 0.5×10^{-3} to 1.

It can be seen directly from (3.10) that the anisotropic deformation factor $\mathcal{A}(\boldsymbol{k})$ reduces to 1 when setting $A_i = 0$, as expected. Moreover, for those modes with wavenumbers $\boldsymbol{k}$ perpendicular to $\boldsymbol{A}$, i.e. $\boldsymbol{A} \cdot \boldsymbol{k} = 0$, it will be shown that $\mathcal{A}(\boldsymbol{k}) = 1$, that is, these modes do not feel the presence of $\boldsymbol{A}$, and thus get no corrections. Modes with $\boldsymbol{A} \cdot \boldsymbol{k} \neq 0$ will get corrections from $\boldsymbol{A}$, but in the limit $\boldsymbol{A} \to 0$ (or more precisely, typical scale of k much larger than A, $k \gg |\boldsymbol{A}|$), it will also be shown that $\mathcal{A}(\boldsymbol{k}) \to 1$, which means that small scales get smaller anisotropic corrections from $\boldsymbol{A}$.

From (3.10),

$$|\mathcal{A}(\boldsymbol{k})|^2 = 1 - \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{c^4k^4}\left[\frac{4}{3}\nu(\nu^2-1) + \frac{\nu(4\nu^2-1)}{3}\epsilon\gamma\right]. \tag{3.11}$$

It is interesting to note that the leading order correction term are quadrupole, there is no dipole correction. Thus, our result gives the anisotropic spectrum of ACW form [4], $P(\boldsymbol{k}) = P(k)\left[1 + g(k)(\hat{n}\cdot\boldsymbol{k})^2\right]$ with

$$g(k) = -\frac{\gamma A^2}{c^4k^2}\left[\frac{4}{3}\nu(\nu^2-1) + \frac{\nu(4\nu^2-1)}{3}\epsilon\gamma\right]. \tag{3.12}$$

In our model, $g(k) < 0$ and it has an apparent k-dependence, $g(k) \sim -1/k^2$. This result has two implications. First, the deviation from statistical isotropy of the power spectrum is *not* constant on all scales (while in [4] and other works it is often assumed that $g(k)$ is a constant g_*). Statistical anisotropy of C_ℓ spectrum mainly appears at the lowest ℓ's, while the spectrum at higher ℓ's remains highly isotropic (this can be seen clearly from FIG.1, where for smaller k, $|\mathcal{A}(\boldsymbol{k})|$ is highly anisotropic, while for larger k, $|\mathcal{A}(\boldsymbol{k})|$ remains flat). Second, for a given direction $\hat{\boldsymbol{k}}$, since $g(k) \sim -1/k^2$, the spectrum itself is suppressed at large scales (see FIG. 2). This phenomenon explains the loss of power at the lowest ℓ's in the C_ℓ spectrum.

Finally, the power spectrum of scalar perturbation on superhorizon scales reads,

$$\begin{aligned}\Delta^2_{\delta\phi}(\boldsymbol{k}) &\equiv \frac{k^3}{2\pi^2}|\delta\phi_{\boldsymbol{k}}|^2 \\ &\simeq \left(\frac{H}{2\pi}\right)^2\left(\frac{ck}{aH}\right)^{3-2\nu}\frac{|\mathcal{A}(\boldsymbol{k})|^2}{cP_{,X}} \\ &= \Delta^2_{\delta\phi}(k)\,|\mathcal{A}(\boldsymbol{k})|^2\,,\end{aligned} \tag{3.13}$$

where $|\mathcal{A}(\boldsymbol{k})|^2$ is given in (3.11). The general anisotropic power spectrum (3.13) is function of k and θ. In our model, its *shape* is similar to that of $|\mathcal{A}(\boldsymbol{k})|^2$ (see FIG. 1).

C. Suppression of Power on the Largest Scales

The observational data appear to suggest a spatial modulation in the CMB spectrum. Especially, a vanishing CMB temperature auto-correlationi on the largest scales has been investigated by several authors [3]. As mentioned before, this can be naturally explained in our model, due to the suppression behavior of $|\mathcal{A}(\boldsymbol{k})|$, $g(k) \sim -1/k^2$, (see FIG. 2 and FIG. 3).

IV. CONCLUSION AND DISCUSSION

In this work, we studied cosmological perturbations in the presence of a superhorizon inhomogeneous inflaton background. We find that the leading-order correction to the standard isotropic power spectrum is a quadrupole anisotropy. Moreover, the deviation from isotropy is generally scale-dependent. More specifically, the quadrupole correction term has a suppression factor $g(k) \sim -1/k^2$. And thus our model can simultaneously explain the observed anisotropic alignment of the low-ℓ multipoles and their low power. If the indication of these large scale CMB anomalies were to be confirmed by future experiments, the work presented here would improve our understanding of inflation and the very early universe.

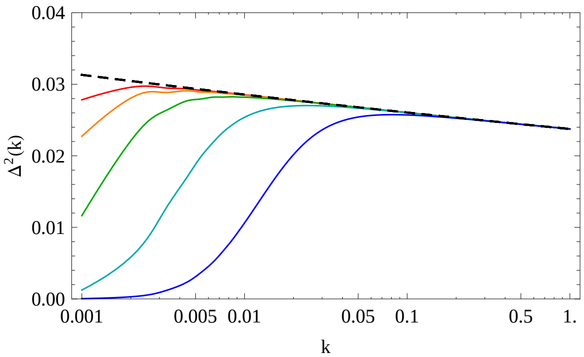

FIG. 3: Suppression of the power on the largest scales (for a given direction $\hat{\boldsymbol{k}}$). The dashed line is traditional isotropic spectrum with $n_s = 0.96$. From top to bottem, $A = 0.3 \times 10^{-3}, 0.5 \times 10^{-3}, 10^{-3}, 0.3 \times 10^{-2}, 10^{-2}$. Parameters are chosen as $\epsilon = 0.01$, $\gamma = H = 1$, $c = 1.2$. The range of k is from 10^{-3} to 1.

In our work, the inhomogeneous background affects the perturbation through the non-linear structure of $P(X, \phi)$ of X. A similar effect would arise when one consider backreaction of superhorizon perturbations through the nonlinearity of the Einstein equations [10].

In our approximations, we neglected the backreaction of the inhomogeneous inflaton background to the geometry, which has to be considered in a more rigorous analysis. In that case, a non-FRW metric may arise. For instance, in [15], anisotropic inflatonary models and perturbations are investigated, where an anisotropic spectrum is also produced. In a FRW background, the quantization procedure is well undersood for canonical variables on sub-Hubble scales, where they evolve adiabatically, and thus the adiabatic vacuum is well-defined. In this work, we choose the mode solution (3.6) by simply noticing that (3.6) reduces to the standard mode solution in the $A_i \rightarrow 0$ limit. It should be noted, however, that the frequency term in the second-line of (3.5) is divergent when $\eta \rightarrow -\infty$, due to the presence of the anisotropic term $\gamma(\boldsymbol{A} \cdot \boldsymbol{k})^2/(aH)^2$. Thus, the presence of a WKB regime and also the quantization procedure in this case are non-trivial in general (see e.g. discussions in [15]). We would like to investigate these problems in future work.

Indeed, the work presented here is a general framework, while the results of our model are generic. A more detailed study including metric perturbations and possible non-FRW background geometry is needed. We would like to present it in a companion work [13].

Acknowledgments

I would like to thank Robert Brandenberger, Miao Li, Tao Wang, Yi Wang, Bin Hu, for useful discussions and comments. I am grateful to Robert Brandenberger for a careful reading the manuscript. This work was supported by Grants of NSFC.

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